A Mode Matching Study of Three Dimensional Microwave Structures Using Eigenfunction Expansions

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Abstract— This project aims at the development of a Three Dimensional (3D) Mode MATCHING Technique for the study of electrically large electromagnetic structures like Reverbaration Chambers or Auto-Focused Microwave Cavities.

I .INTRODUCTION

From a different point of view this effort aims at the establishment of an analytical eigen-function expansion technique able to handle electrically large structures enclosing relatively small objects with complicated inhomogeneous synthesis. The task of this project is to employ a numerical-scheme discretizing only the E/M field in relatively small objects inside large chambers, while the truly large surrounding space will be described with the aid of an eigenfuction expansion. The binding of the numerical field description inside the object to the surrounding space analytical field expansion will be implement through equivalent electric and magnetic current surface densities defined on the surface. The latter will be defined based on the Love's field equivalence principle. Since the implementation of this idea involves numerous difficulties an analytical prove of concept will be first elaborated. Thus the present effort is concentrated on these analytical formulations. In view of the above, the idea to be tested herein is as follows, consider a canonically shaped (cavity) closed domain like a paralellepid Reverbaration chamber with known analytically Eigenfunction. As a first step of this project we assume a waveguide with circular cross section, which is playing the role of the empty reverberation chamber.

Working toward this task, the field expressions inside a circular waveguide filled with two concentric dielectric layers is first reviewed. These are exploited to define equivalent electric and magnetic current densities on the cylindrical interface of the two media. Adopting Love's equivalent principle these equivalent current densities must yield the same fields as these of the analytical solution. Explicitly, the resulting equivalent currents on the cylindrical shell are considered in the same waveguide homogeneously filled with the outer dielectric layer. These fields are matched to the shell current densities. In turn the orthogonality properties of the TE&TM modes are exploited to isolate the weighting factors of the expansion. The resulting field should match to the original inhomogeneous waveguide modal field distribution.

II. MATH

Taking under consideration this geometry for the waveguide the analytical equations for the electric and magnetic field in its interior are extracted. For a general approach the analytical expressions for the E/M fields inside a circular cross section waveguide which contains two different dielectrics are determined. The inner medium plays the role of the object inside the reverberation chamber and the outer medium the large empty space. Examining this geometry the analytical expressions for the E/M fields were derived. In that case hybrid modes, HE and EH, were excited inside this structure due to the conjugation of the two materials and orthogonal modes only in specific case where p=0. Also the characteristic equation of that particular geometry was exported.

$$\left\{ \frac{\varepsilon_1 k_2}{\varepsilon_2} J_p'(k_1 b)W - k_1 J_p(k_1 b)X \right\} \left\{ k_2 J_p'(k_1 b)Y - k_1 J_p(k_1 b)Z \right\} -$$

$$- \frac{p^2 \bar{\beta}^2 (k_1^2 - k_2^2)^2}{\varepsilon_2^2 k_1^2 k_2^2 b^2} J_p^{\ 2}(k_1 b)WY = 0$$

$$\bar{\beta} = \frac{\beta}{\omega \sqrt{\mu_0 \varepsilon_0}} \quad : \text{Normalized phase constant}$$

$$W = \begin{vmatrix} J_p(k_1 a) & Y_p(k_1 a) \\ J_p(k_1 b) & Y_p(k_1 b) \end{vmatrix} \qquad X = \begin{vmatrix} J_p(k_1 a) & Y_p(k_1 a) \\ J_p'(k_1 b) & Y_p'(k_1 b) \end{vmatrix}$$

$$Y = \begin{vmatrix} J'_p(k_1a) & Y'_p(k_1a) \\ J_p(k_1b) & Y_p(k_1b) \end{vmatrix} \quad Z = \begin{vmatrix} J'_p(k_1a) & Y'_p(k_1a) \\ J'_p(k_1b) & Y'_p(k_1b) \end{vmatrix}$$

W,X,Y,Z: Wronskians

Because it is very hard to solve the characteristic equation analytically in order to give a closed form a program, at Matlab environment, was built to solve it. The program was returning the solution of the equation, for the particular structure, which is the eigenvalue β. The correctness of the equations, and also the program thaw a made, confirmed through the existence of backward waves and the comparison between the bibliography and the program diagrams.
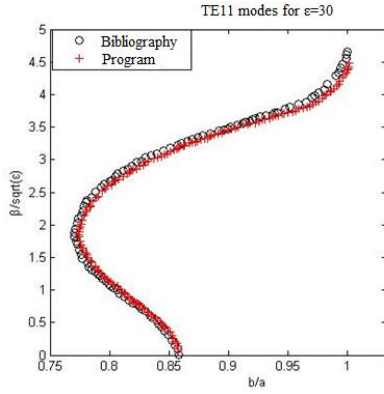
The binding of these two geometries was made through the equivalent electric and magnetic current densities defined in the surface, using Love's field equivalence principle.

$$\hat{n} \times \bar{H}_1 = \bar{J}_s \quad \text{and} \quad \bar{E}_1 \times \hat{n} = \bar{M}_s$$

Having in mind the analytical expressions for the E/M fields inside the waveguide which contains two dielectrics we derive the analytical expressions of surface electric and magnetic current densities in the surface that separates the two media.. These current densities are describing the inner medium, so the idea is that we can solve an equivalent problem ignoring the inner medium, so we apply them at a hypothetical surface at the waveguide with one medium inside. For the sake of generosity in that method we assume that the E/M fields that are generating from these currents are unknown therefore will considered as a sum of all the modes multiplied by a weighting factor.

$$\bar{H}_\varphi = \hat{\varphi} \cdot \sum_{p=1}^{\infty} W^\varphi_{p_m} \cdot h_{\varphi_p} \quad , \quad \bar{H}_z = \hat{z} \cdot \sum_{p=1}^{\infty} W^z_{p_m} \cdot h_{z_p}$$

$$\bar{E}_\varphi = \hat{\varphi} \cdot \sum_{p=1}^{\infty} W^\varphi_{p_e} \cdot e_{\varphi_p} \quad , \quad \bar{E}_z = \hat{z} \cdot \sum_{p=1}^{\infty} W^z_{p_e} \cdot e_{z_p}$$

Where W are the weighting factors

The final task of this project was to export that weighting .In order to achieve that we use the orthogonality principle of the modes to isolate them from the summation. The two most effective approaches in order to isolate the weighting factors is the "Poynting approach", where we multiply the magnetic field external to with the electric field and the opposite, and the "Arbitrary approach", where we chose the base function which we will multiply in a way to help us solve the equations and find the weighting factors in the easiest way. Using these approaches we derived the weighting factors in such a way that can be calculated. Solving all the integrals and doing all the necessary mathematics we export a set of equations for the weighting factors like the above:

$$-\frac{b^2}{2} B \cdot J_p(k_2b) J_p(kb) \cdot N_{pq}^{e,z,TM} \cdot \left(\pi + \frac{1}{4p} \sin 4\pi p \right) =$$
$$= W_{vm} \cdot N_{vq}^{h,z,TE} \cdot N_{pq}^{e,z,TE} \cdot \left(\pi + \frac{1}{4v} \sin 4\pi v \right) \cdot$$
$$\cdot \frac{a^2}{2} \left\{ J_v'^2(ka) + \left[1 - \frac{(v)^2}{(ka)^2} \right] J_v^2(ka) \right\}$$

It can be seen that in some equations the weighting factors are equal to zero, so that group of modes does not being excited from these particular currents.

III. Next Career Plans

My plan for the future is to pursue as a PhD at the Biotechnology area in order to help other people with my work and make their lives more easy, safe and healthy.

The fact that I was selected for the MTT-S award gave me strength to work even harder than before and keep chasing my dreams. I hope to have the opportunity to be selected again in the future for more awards from the IEEE community. Furthermore the MTT-S scholarship gave me the opportunity to be informed about all the technological development through the monthly magazines and expand my horizons.