Sub-Nyquist Sampling for Software Defined Radios

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***Abstract*— This report presents the investigation of the application of two different types of sub-Nyquist sampling schemes, undersampling and periodic non-uniform sampling, to RF receiver architectures. Off the shelf evaluation kits were used to explore how undersampling ADCs work and how undersampling affects the demodulation of AM waveforms. Then a small lows speed periodic non-uniform sampler was designed to act as a real time FM radio receiver.**

***Index Terms*— Periodic Non-uniform Sampling (PNS), Analog-to-Digital Converter (ADC), sub-Nyquist sampling**

I. INTRODUCTION

Historically, most mixed signal receiver architectures followed the classical interpretation of the Nyquist-Shannon sampling theorem: A signal can be 'perfectly' reconstructed if it is sampled uniformly at twice its highest frequency. In practice, this meant that a mixed signal receiver required an extensive analog frontend to mix down a received signal before it could be sampled by an analog-to-digital converter (ADC).

However, since the formation of the Nyquist-Shannon sampling theorem in the early 20th century, ongoing research in the fields of telecommunication and signals and systems has shown that a signal does not need to be sampled uniformly [1] nor does it need to be sampled at twice the signal's highest frequency to be perfectly reconstructed. Instead, good reconstruction is possible as long as the average sampling rate, either uniformly or non-uniformly distributed, is twice as fast as the highest bandwidth of interest [2].

Most modern communication systems already apply sub-Nyquist sampling as society's demand for increased bandwidth has outpaced the development of higher rate, low cost ADCs,.

The objective of my research was to investigate how different sub-Nyquist sampling schemes could be used in receiver architectures (1) to simplify the analog circuitry in the receiver frontend and (2) to increase the re-configurability of the receiver, while maintaining similar levels of performance to traditional receiver architectures [3].

II. METHODOLOGY

In pursuit of this objective, I investigated two different methods of sub-Nyquist sampling, undersampling and periodic non-uniform sampling (PNS), for their use in direct conversion receiver architectures. I chose these two sub-Nyquist sampling methods because they represent two of the most basic and fundamental sub-Nyquist sampling schemes.

The first method, undersampling, uses aliasing, in combination with very sharp bandpass filters, to fold the frequency spectrum of interest into the baseband frequency range, performing direct conversion at the sampling stage. The second method, periodic non-uniform sampling, like its namesake, uses a non-uniform periodic sampling pattern, in combination with digital stitching, to perform direct down conversion of received signal without distorting the phase of the message signal. Both sampling schemes have various trade-offs similar to homodyne and heterodyne receivers. Simple undersampling receivers, like homodyne receivers, are sensitive to phase differences between the transmitter and receiver carrier signals. This sensitivity limits the ease at which higher order modulation schemes can be used in an undersampling system. PNS receivers are not phase sensitive, similar to heterodyne receivers, but require complicated stitching filters after sampling to reconstruct the message signal without aliasing [4].

I investigated undersampling by examining the use and application of two undersampling ADCs evaluation modules (EVMs), AD6649-EVM and AD9645-EVM, made by Analog Devices. The AD6659 boasts a sampling rate of 250 MSPS with an analog input bandwidth of 400 MHz. The AD9645 has a sampling rate of 125 MSPS with a claimed analog input bandwidth at full power of 650 MHz.

First, I measured a wideband frequency response of both the ADCs to determine how much each ADC could undersample and to see the practical consequences of undersampling. Second, I examined the effects of undersampling on modulated signals by using the AD6649-EVM to undersample an AM signal which I then demodulated in MATLAB. In the future, I will further examine how undersampling distorts encoded information by using the AD6649-EVM to undersample signals with higher order modulation schemes.

I investigated PNS by designing a mixed signal FM band receiver frontend for a DE0-nano Altera FPGA development board. The mixed signal frontend performs periodic non-uniform sampling by clocking two ADCs together with a small deliberate clock skew generated by a digital delay line.

This work was supported in part by a scholarship given by the Microwave Theory and Techniques Society.

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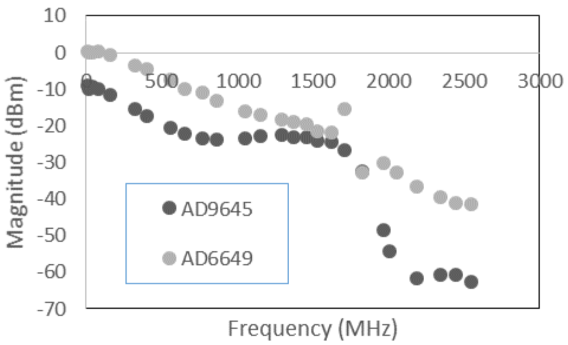

Fig. 1. Frequency response of the undersampling ADC-EVMs stitched back together from the aliased frequency response

The entire receiver fits on a small two layer PCB and uses relatively inexpensive components. After I soldered the board together, I used a function generator as a clock to confirm that the ADCs were sampling non-uniformly. In the future, I will finish interfacing the Altera DE0-nano to the frontend so that the Altera FPGA can collect the ADC samples and digitally stitch the message signal back together in real time.

III. RESULTS

A. *Characterization of Undersampling Evaluation Modules*

As described previously, we measured the frequency response of the evaluation modules to determine how much we could undersample signals before the signals became unrecoverable. Shown in fig. 1, both of the ADCs offer 'good' performance until approximately 1.8 GHz. After that, the ADD6649 ADC EVM offered better performance than the AD9645 ADC EVM.

After measuring the frequency response of the ADCs, we used the AD6649-EVM to undersample an AM signal created by modulating a 1.15 GHz tone with a 100 kHz tone. The AM signal was sampled at 250 MSPS or with an undersampling ratio of 4. The undersampling operation mixed the 1.15 GHz tone down to 100 MHz. At this point, to successfully demodulate the AM signal, we wrote a MATLAB script to identify the frequency and phase of the carrier wave. This information was used to generate a tone that was then mixed with the undersampled AM signal which was then filtered to pull out the 100 KHz message signal. Shown in Fig. 2 is the frequency spectrum of the received signal after it has been mixed down by the effects of undersampling and by the deliberate digital down-conversion.

B. *Periodic Non-Uniform Sampling Frontend*

After I soldered the board together, I confirmed that the two AD830 ADCs sampled with a set phase delay using a function generator. Shown in Fig. 3 is the top side of the PCB with an edge connector designed to interface to the DE0-nano kit. I plan on interfacing the board with a DE0-nano in the downtime of the upcoming summer months.

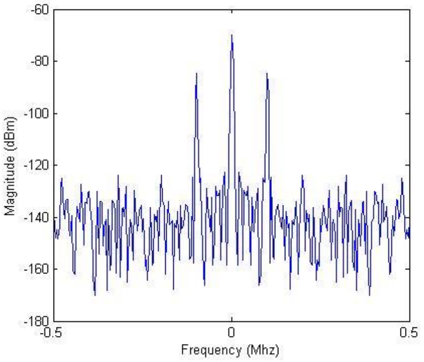

Fig. 2. Frequency Spectrum of the received AM signal after sampling and digital down conversion.

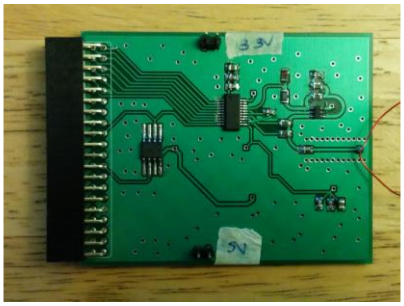

Fig. 3. PNS receiver printed circuit board with limited analog frontend circuitry

IV. ACKNOWLEDGEMENTS AND FUTURE PLANS

Overall, I found the study of sub-Nyquist sampling systems to be very compelling. I would like the give special acknowledgement to MTTS for their undergraduate research scholarship because it had a significant impact on choices I made in undergraduate education. Specifically, undergraduate research helped me determine what subject areas I want to pursue in graduate. In particular, I discovered a passion for the design of mixed signal systems during my research paper review. I feel that, even though my research was limited in scope, the research supported by MTTS gave me a competitive edge when applying to graduate schools.

In the fall, I am pursuing a PhD in Electrical Engineering at Stanford University. I am interested in learning more about the design of mixed signal systems for communication systems, biomedical, agricultural and biological applications.

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