

# Below-Cutoff Propagation in Metamaterial-Lined Circular Waveguides

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**Abstract**—This report presents the intriguing phenomena associated with below-cutoff propagation in circular waveguides whose interior surface is coated with a thin metamaterial liner possessing dispersive, negative, and near-zero permittivity. It is shown that such systems offer the potential for miniaturized waveguide components suitable for applications in which the waveguide volume must remain largely empty.

**Index Terms**—Metamaterials, circular waveguides, epsilon-near-zero, negative permittivity, backward-wave, below-cutoff propagation, miniaturization,

## I. INTRODUCTION

HOLLOW waveguides are widely used in applications requiring high power-handling capability and simple integration with radiating devices, such as horn antennas. However, their size being dictated by their operating frequency places them at a disadvantage in the low frequency regime. While homogeneously filling the vacuum region of the waveguide serves to reduce their cutoff frequencies, it is sometimes desirable to have access to the enclosed region of a waveguide, making it convenient to partially fill it with a dielectric to achieve similar results. This report presents the most significant results of a study on using thin-liners composed of metamaterials to enable waveguide miniaturization, in which details can be found in our published works [1]. Potential applications of this study include the design of metamaterial-lined waveguides to demonstrate reverse Cherenkov radiation, as in Ref. [2], and the inclusion of metamaterials inside magnetic-resonance scanners to enable traveling-wave imaging at low static field strengths, as proposed in Ref. [3].

## II. RESULTS

Figure 1(a) presents the geometry of the metamaterial-lined PEC circular waveguide under consideration whose physical dimensions are  $b = 15\text{mm}$  and  $a = 14\text{mm}$  (liner thickness of  $t = 1\text{mm}$ ) with an inner vacuum region (relative parameters  $\epsilon_{r1} = \mu_{r1} = 1$ ) and a liner region with dispersive  $\epsilon_{r2}$  and a nonmagnetic response ( $\mu_{r2} = 1$ ). The dispersive nature of  $\epsilon_{r2}$  is described by a lossless Drude model with  $\epsilon_{r2}(\omega) = 1 - \omega_{ep}^2/\omega^2$ , in which  $\omega_{ep} = 3.5497\text{GHz}$  is the plasma frequency.

In Fig. 1(b), the dispersion of the metamaterial-lined waveguide's  $HE_{11}$  mode is shown alongside the  $TE_{11}$ -mode dispersion of a homogeneously vacuum-filled (unlined) waveguide of the same outer dimensions. Comparing both, the introduction of a liner only significantly alters the dispersion in the frequency range in which the metamaterial takes on negative, near-zero permittivity. Accordingly, in this regime, the metamaterial-lined waveguide has a frequency-reduced

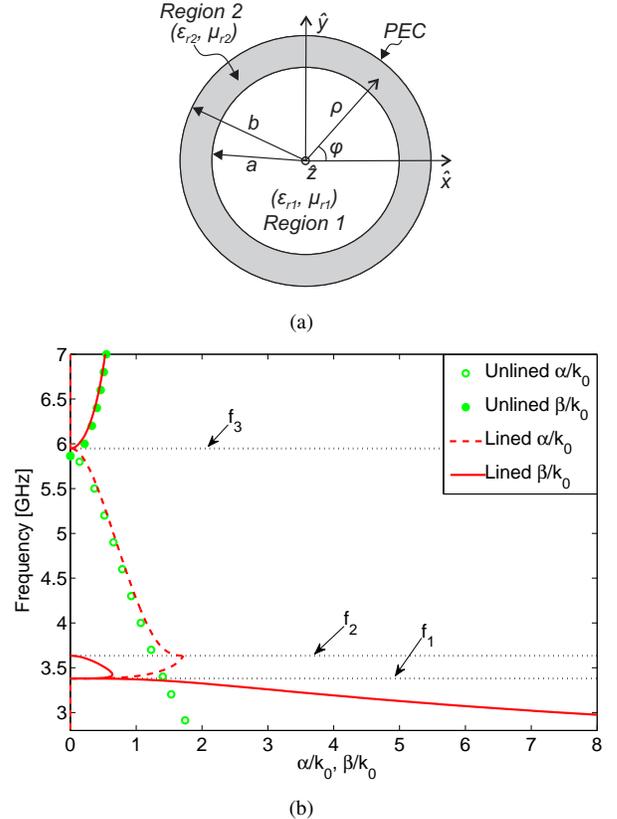


Fig. 1. (a) Transverse cross-section of the metamaterial-lined circular waveguide. (b) The  $HE_{11}$ -mode cutoff frequency versus  $\epsilon_{r2}$  (red solid line). The permittivity of the liner  $\epsilon_{r2}(\omega)$  (green dashed line) follows a Drude model with  $\omega_{ep} = 3.597\text{GHz}$ .

band that is well below the cutoff of the unlined waveguide. This is composed of a propagating backward-wave band for  $f \leq f_1 = 3.381\text{GHz}$  and a complex propagating band for  $f_1 < f < f_2 = 3.644\text{GHz}$ . Subsequently, the dispersion and attenuation resembles that in an unlined waveguide and there exists a stopband for  $f_2 \leq f \leq f_3 = 5.958\text{GHz}$  followed by a forward-wave band for  $f > f_3$ .

Using the full-wave simulation software HFSS [4] on the simulation model shown in Fig. 2(a), a transmission analysis is performed. In this representative setup, two larger vacuum-filled circular waveguides with a radius  $b_{large} = 30\text{mm}$  and a  $TE_{11}$  cutoff frequency of  $2.928\text{GHz}$  are connected by a smaller vacuum-filled waveguide with a radius  $b_{small} = 15\text{mm}$  and a  $TE_{11}$  cutoff frequency of  $5.857\text{GHz}$ . A waveport located at the end of one of the larger waveguide sections excites the  $TE_{11}$  mode at frequencies that lie in the propagating region

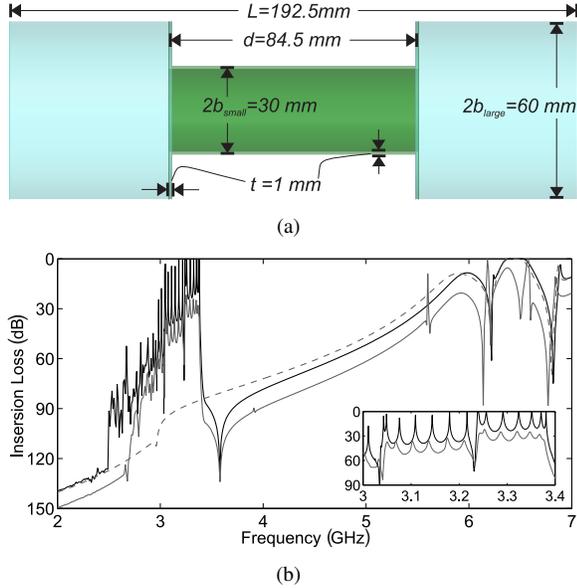


Fig. 2. (a) Full-wave simulation model employed in the transmission analysis. (b) Insertion loss for the unlined case (dashed-grey curve), the lined case with no loss (solid-black curve) and the lined case with loss (solid-grey curve). The inset shows in detail several resonances in the frequency-reduced backward-wave passband.

of the larger waveguide, but which correspond to the natural evanescent region of the smaller waveguide. The dashed grey curve in Fig. 2(b) presents the insertion loss obtained for this setup and verifies that the intermediate waveguide under cutoff strongly attenuates the  $TE_{11}$  mode.

Now, a metamaterial liner of thickness  $t = 1\text{mm}$  is introduced into the smaller waveguide and assigned the dispersive permittivity,  $\epsilon_{r2}(\omega)$ . Inside the frequency-reduced band, the metamaterial liner would effectively enable a cross-sectional-area reduction of the unlined circular waveguide by a minimum of 75%. To understand the impact of losses in the metamaterial liner on the ability of the lined waveguide to transport power, both lossy ( $\omega_t = 5\text{MHz}$ ) and lossless ( $\omega_t = 0$ ) cases are compared. Figure 2(b) shows the insertion loss in the lossless (solid black curve) and lossy (solid grey curve) cases, in which the peaks of transmission in the backward-wave band are shown in the inset. When the liner is lossless, these transmission peaks achieve total transmission of power through the lined waveguide section. Since the fields are concentrated strongly in the liner region, the introduction of loss degrades the transmission through the structure; nevertheless, the backward-wave band introduced by the liner exhibits a dramatic increase in transmission over the unlined case, at times showing enhancements of over 56dB. In fact, even in the lossy case, the transmission attains a peak value of  $-21.7\text{dB}$  at  $f = 3.253\text{GHz}$ .

Investigations are currently underway to realize these thin-metamaterial liners through using radial and azimuthal arrays of thin-wires loaded with discrete inductors, in which a representative design is shown in Fig. 3. Although preliminary studies reveal that these type of structures can support below-cutoff propagation akin to the isotropic metamaterial liners observed before, ongoing work is focused on determining the

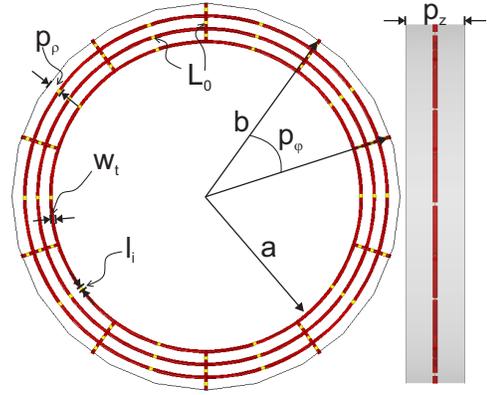


Fig. 3. Front and side views of the thin-wire lined circular waveguide consisting of an inner core region of radius  $a$  and a metamaterial liner of thickness  $b - a$ .

implications of their anisotropic material parameters. Nevertheless, the thin-wire structure is a viable candidate for experimentation, and, as such, current work is being made towards their fabrication.

### III. FUTURE CAREER PLANS

I hope to apply my broad range of engineering design skills and a highly specialized expertise in metamaterial applications that have been developed in my PhD programs towards a start-up tech company. This company would market below-cutoff propagation in metamaterial-lined waveguides towards applications in medical imaging, sensing, communications, and security. Through the MTT-S Graduate Fellowship, I was capable of being highly productive throughout the year without having to overly worry about personal finances. That enabled me to investigate numerous practical applications of my early theoretical work. I have been privileged to attend IMS in 2011, 2012, and 2013, and must say that each trip has changed me and my outlook on research. The opportunity to discourse with like-minded individuals from around the world has helped cement myself in the microwave community. Furthermore, I am truly grateful to have met the rock stars of the microwave field and hear first-hand their latest research experiences. For all of this, I am eternally in the debt of the MTT-S society who has helped me become the researcher that I am today.

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